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A receiver for identifying a message based upon a received signal, the receiver comprising:

a processor that generates a minimum threshold and a maximum threshold representing a variable range for each of a plurality of possible message levels, and

a comparator that identifies the message by comparing the received signal with the generated minimum and maximum thresholds.

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1 5.) The receiver according to claim 1, wherein the generated minimum and maximum
2 thresholds define a range wherein the probability of correctly receiving a selected
3 signal exceeds a selected probability P_0 .

1 6.) The receiver according to claim 5, wherein the processor includes a means for
2 calculating the mean value, $Lev(i)$, within a selected range defined by a selected
3 set of minimum and maximum thresholds.

1 7.) The receiver according to claim 6, wherein the processor includes a means for
2 calculating a distance $d(i)$ between received signal levels, the distance $d(i)$ being
3 calculated according to the equation:

$$d(i) = Lev(i+1) - Lev(i) - Lmse(i+1) - Lmse(i),$$

4
5 wherein the term "i+1" identifies a message level adjacent the i^{th} message
6 level in the constellation design for the receiver and wherein $Lmse(i)$ is the level
7 mean square error for the i^{th} message level.

1 8.) The receiver according to claim 7, wherein the distance $d(i) > d_{min}$ for all message
2 levels.

1 9.) The receiver according to claim 1, wherein the processor includes means for
2 determining a distance $d(i)$ between received signal levels, the distance $d(i)$
3 having different values for a plurality of message levels.

10.) A method of forming a constellation design having a selected number of (i)
message levels, the constellation design forming part of a receiver that identifies a
transmitted message based upon a received signal, the method comprising:
determining a minimum threshold and a maximum threshold representing
a variable range for each of a plurality of possible signal levels, and
calculating the distance $d(i)$ between possible signal levels based upon the
determined minimum and maximum thresholds.

11.) The method according to claim 10, wherein the determining step comprises the
steps of:
identifying a probability density function for each possible signal level Y ,
and
identifying the minimum and maximum thresholds as the boundaries of a
range in the identified probability density function wherein the probability of
correctly receiving a selected message level exceeds a selected probability P_0 .

12.) The method according to claim 11, wherein the step of identifying the probability
density function comprises the steps of:
transmitting data points to the receiver, and
recording the received signal level associated with each of the transmitted
data points.

1 13.) The method according to claim 10, wherein the step of calculating the distance
2 $d(i)$ between received signal levels further includes the steps of:
3 determining the mean value, $Lev(i)$, for a selected variable range
4 identified by a selected set of minimum and maximum thresholds, and
5 calculating the distance $d(i)$ as a function of $Lev(i)$.

1 14.) The method according to claim 13, further including the step of calculating the
2 distance $d(i)$ in accordance with the equation:
3
$$d(i) = Lev(i+1) - Lev(i) - Lmse(i+1) - Lmse(i);$$

4 wherein the term "i+1" identifies a message level adjacent the i^{th} message level
5 in the constellation design for the receiver and wherein $Lmse(i)$ is the level mean
6 square error for the i^{th} message level.

1 15.) The method according to claim 13, further comprising the step of identifying
2 whether the calculated distance $d(i) > d_{min}$, wherein d_{min} represents a selected
3 minimum value.

1 16.) The method according to claim 15, further comprising the step of adjusting the
2 constellation design such that the distance $d(i) > d_{min}$ for all received signal levels
3 in the constellation design.

1 17.) The method according to claim 12, further comprising the step of calculating the
2 mean value, $Lev(i)$, according to the equation:

$$Lev(i) = \frac{1}{N} \sum_{i=1}^N L(i),$$

wherein $L(i)$ is the training data received by the receiver, and

N is the number of times training data for the i^{th} level is sent.

- 18.) The method according to claim 17, further comprising the step of calculating the standard mean square error, σ^2 , according to the equation:

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N [L(i) - Lev(i)]^2.$$

- 19.) The method according to claim 18, further comprising the step of calculating the $Lmse(i)$ according to the equation:

$$Lmse = \alpha \sigma^2,$$

where α meets the following

$$P0 = \frac{\int_{-\infty}^{\infty} e^{\frac{-x^2}{2\sigma^2}} dx}{\int_{-\infty}^{\infty} e^{\frac{-x^2}{2\sigma^2}} dx}, \text{ where } P0 \text{ is a selected probability.}$$

- 20.) A receiver for identifying a transmitted message based upon a received signal, the receiver comprising:

a processor for generating a constellation design having a minimum threshold and a maximum threshold for each of a plurality of possible signal levels, the minimum and maximum thresholds representing a variable range that may differ between possible signal levels, and

7 a comparator that identifies the transmitted message by comparing the
8 received signal with the generated constellation design and that generates an
9 output signal representative of the transmitted message.

1 21.) A method of identifying a message based upon a received signal, the method
2 comprising:
3 receiving the signal,
4 generating a minimum threshold and a maximum threshold representing a
5 variable range for each of a plurality of possible message levels, and
6 identifying the message by comparing the received signal with the
7 generated minimum and maximum thresholds.

1 22.) The method according to claim 21, wherein the minimum and maximum
2 thresholds are generated as a function of an interrelationship between noise and
3 the message level.

1 23.) The method according to claim 22, wherein the minimum and maximum
2 thresholds are generated as a function of the interrelationship between digital
3 impairment and the message level.

1 24.) The method according to claim 22, wherein the minimum and maximum
2 thresholds are generated as a function of the interrelationship between coherent
3 noise and the message level.

Sub 25.1

25.) The method according to claim 21, wherein the generating step includes the step of calculating a variable range $Lmse(i)$ for each possible message level Y , $Lmse(i)$ representing one-half the distance between the minimum and maximum thresholds for each possible message level, wherein the minimum and maximum thresholds define a range wherein the probability of correctly receiving a selected signal exceeds a selected probability P_0 .

26.) The method according to claim 25, further including the step of calculating the mean value, $Lev(i)$, within a selected range defined by a selected set of minimum and maximum thresholds.

27.) The method according to claim 26, further including the step of calculating a distance $d(i)$ between received signal levels, the distance $d(i)$ being calculated according to the equation:

$$d(i) = Lev(i+1) - Lev(i) - Lmse(i+1) - Lmse(i).$$

28.) The method according to claim 21, further including the step of determining a distance $d(i)$ between received signal levels, the distance $d(i)$ having different values for a plurality of message levels.

29.) The method according to claim 28, wherein the step of calculating the distance $d(i)$ between received signal levels further includes the steps of:

3 determining the mean value, $Lev(i)$, for a selected variable range
4 identified by a selected set of minimum and maximum thresholds, and
5 calculating the distance $d(i)$ as a function of $Lev(i)$.

1 30.) The method according to claim 28, further comprising the step of identifying
2 whether the calculated distance $d(i) > d_{min}$, wherein d_{min} represents a selected
3 minimum value.

1 31.) The method according to claim 30, further comprising the step of adjusting the
2 constellation design such that the distance $d(i) > d_{min}$ for all received signal levels
3 in the constellation design.

1 32.) The method according to claim 21, wherein the generating step comprises the
2 steps of:
3 identifying a probability density function for each possible signal level Y ,
4 and
5 identifying the minimum and maximum thresholds as the boundaries of a
6 range in the identified probability density function wherein the probability of
7 correctly receiving a selected message level exceeds a selected probability P_0 .

1 33.) The method according to claim 32, wherein the step of identifying the probability
2 density function comprises the steps of:
3 transmitting data points to the receiver, and

4 recording the received signal level associated with each of the transmitted
5 data points.

1 34.) The method according to claim 33, further comprising the step of calculating the
2 mean value, $Lev(i)$, according to the equation:

$$3 \quad Lev(i) = \frac{1}{N} \sum_{i=1}^N L(i),$$

4 wherein $L(i)$ is the training data received by the receiver, and
5 N is the number of times training data for the i^{th} level is sent.

1 35.) The method according to claim 34, further comprising the step of calculating the
2 standard mean square error, σ^2 , according to the equation:

$$3 \quad \sigma^2 = \frac{1}{N} \sum_{i=1}^N [L(i) - Lev(i)]^2.$$

1 Sub A196.) The method according to claim 35, further comprising the step of calculating the
2 $Lmse(i)$ according to the equation:

$$3 \quad Lmse = \alpha \sigma^2,$$

4 where α meets the following

$$5 \quad P0 = \frac{\int_{-\infty}^{\infty} e^{\frac{-x^2}{2\sigma^2}} dx}{\int_{-\infty}^{\infty} e^{\frac{-x^2}{2\sigma^2}} dx}, \text{ where } P0 \text{ is a selected probability.}$$